A NOTE ON FUZZY REAL AND COMPLEX FIELD

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Abstract. Using the concept of fuzzy field, we have considered the fuzzy field of real and complex numbers and thereafter we have established a few standard results of real and complex numbers with respect to a membership function.

Key Words : Fuzzy number , Membership function , Fuzzy Field .

Introduction : The concepts of Fuzzy real and complex numbers; fuzzy functions; limit, continuity, differentiability, integrability of fuzzy functions; sequence of fuzzy numbers and their convergence has been developed in different papers [1 − 5]. All these concepts has been developed without considering fuzzy field of real or complex numbers. But, in crisp system, considering the set of real or complex numbers as the field of real or complex numbers, usual mathematical analysis has been developed. In the paper [2], S. Nanda, introduced the concept of fuzzy field. Using this concept of fuzzy field, we have considered the fuzzy field of real and complex numbers and thereafter we have established a few standard results of real and complex numbers with respect to a membership function. In particular, if we consider the characteristic function instead of membership function, our results will coincide with the usual results of real and complex numbers. In the first section, we have introduced the fuzzy field of real numbers $\mathbb{R}_\mu$, modulus of a member of $\mathbb{R}_\mu$, supremum , infimum of a subset of $\mathbb{R}_\mu$ and a few properties depending on these concepts. In section two, we have introduced the concepts of sequence in $\mathbb{R}_\mu$, convergence and divergence of a sequence in $\mathbb{R}_\mu$. In section three , we have introduced the concept of fuzzy field of complex numbers $\mathbb{C}_\mu$, complex conjugate, modulus, argument of a member of $\mathbb{C}_\mu$, exponential function, logarithm function in $\mathbb{C}_\mu$ and a few properties depending on these concepts. Lastly we have given a conclusion, where we have described our future research.

Definition 1. [2] *Let X be a field and F a fuzzy set in X with membership function μ_F. Then F is a fuzzy field in X iff. the following conditions are satisfied :*

(i) $$\mu_F(x + y) \geq \min\{\mu_F(x), \mu_F(y)\} \quad for\ all\ x, y \in X$$

(ii)	$\mu_F(-x) \geq \mu_F(x)$	$for\ all\ x\ \varepsilon\ X$
(iii)	$\mu_F(xy) \geq \min\{\mu_F(x), \mu_F(y)\}$	$for\ all\ x, y\ \varepsilon\ X$
(iv)	$\mu_F(x^{-1}) \geq \mu_F(x)$	$for\ all\ x\ \varepsilon\ X$
(v)	$\mu_F(0) = 1 = \mu_F(1)$	

Throughout our discussion , we will consider the field X as the field of real numbers $\mathbb{R}$ or as the field of complex numbers $\mathbb{C}$. Also, we consider $\mathbb{R}_\mu$ as a fuzzy field of real numbers and $\mathbb{C}_\mu$ as a fuzzy field of complex numbers and the members are respectively called $\mu-$ fuzzy real numbers and $\mu-$ fuzzy complex numbers.

Definition 2. *For* $x_1, x_2\ \varepsilon\ \mathbb{R}_\mu$, $x_1 \leq_\mu x_2$ *if and only if* $x_1\mu(x_1) \leq x_2\mu(x_2)$. *Similarly , we can define* $x_1 <_\mu x_2$, $x_2 \geq_\mu x_1$, $x_2 >_\mu x_1$.

Property 1. *Let* $a, b\ \varepsilon\ \mathbb{R}_\mu$. *Then* (i) $a \geq 0 \implies a \geq_\mu 0$ *and* $a \leq 0 \implies a \leq_\mu 0$, (ii) $0 \leq_\mu a \implies -a \leq_\mu 0$, (iii) $0 \leq_\mu a, 0 \leq_\mu b \implies 0 \leq_\mu a + b$, (iv) $a \leq_\mu 0, b \leq_\mu 0 \implies a + b \leq_\mu 0$, (v) $0 \leq_\mu a, 0 \leq_\mu b \implies 0 \leq_\mu ab$, (vi) $a \leq_\mu 0, b \leq_\mu 0 \implies 0 \leq_\mu ab$, (vii) $0 \leq_\mu a, b \leq_\mu 0 \implies ab \leq_\mu 0$, $(viii)$ $a \neq 0 \implies 0 \leq_\mu a^2$.

Definition 3. *Let* $a\ \varepsilon\ \mathbb{R}_\mu$. *Then the modulus of* a *in* $\mathbb{R}_\mu$ *is denoted by* $|a|_\mu$ *and defined as* $|a|_\mu = |a|\mu(a)$, *where* $|a|$ *denotes the usual modulus of* $a\ \varepsilon\ \mathbb{R}$.

Property 2. (i)
$$|a|_\mu = \begin{cases} a\,\mu(a) & if \quad a > 0 \\ 0 & if \quad a = 0 \\ -a\,\mu(a) & if \quad a < 0 \end{cases}$$

Property 3. (ii) $|-a|_\mu = |a|_\mu$, (iii) $\frac{|ab|_\mu}{\mu(ab)} = \frac{|a|_\mu}{\mu(a)}\frac{|b|_\mu}{\mu(b)}$ *provided* $\mu(a)$, $\mu(b)$, $\mu(ab) \neq 0$, (iv) $\frac{|a+b|_\mu}{\mu(a+b)} \leq \frac{|a|_\mu}{\mu(a)} + \frac{|b|_\mu}{\mu(b)}$ *provided* $\mu(a)$, $\mu(b)$, $\mu(a+b) \neq 0$, (v) *Let* $c > 0$, $|a|_\mu < c \implies -\frac{c}{\mu(a)} < a < \frac{c}{\mu(a)}$, $\mu(a) \neq 0$ *and* $|a| <_\mu c \implies -\frac{c\mu(c)}{\mu(a)} < a < \frac{c\mu(c)}{\mu(a)}$, $\mu(a) \neq 0$

Definition 4. *Let* $A \subseteq \mathbb{R}_\mu$. A *is said to be* $\mu-$ *bounded above in* $\mathbb{R}_\mu$ *if the set* $\{x\mu(x) : x\ \varepsilon\ A\}$ *is bounded above in* $\mathbb{R}$. *The* $\mu-$ *supremum of* A *is denoted by* $sup_\mu A$ *and defined by* $sup_\mu A = sup\{x\mu(x) : x\ \varepsilon\ A\}$.

Similarly , A *is said to be* $\mu-$ *bounded below in* $\mathbb{R}_\mu$ *if the set* $\{x\mu(x) : x\ \varepsilon\ A\}$ *is bounded below in* $\mathbb{R}$. *The* $\mu-$ *infimum of* A *is denoted by* $inf_\mu A$ *and defined by* $inf_\mu A = inf\{x\mu(x) : x\ \varepsilon\ A\}$.

A set $A \subseteq \mathbb{R}_\mu$ *is said to be* $\mu-$ *bounded if it is both* $\mu-$ *bounded above and* $\mu-$ *bounded below.*

Theorem 1. *If $A \subseteq \mathbb{R}$ is bounded then it is $\mu-$ bounded.*

Proof. Obvious. □

Completeness property of $\mathbb{R}_\mu$:

Statement I : Every non - empty $\mu-$ bounded above subset of $\mathbb{R}_\mu$ has a $\mu-$ supremum.

Statement II : Every non - empty $\mu-$ bounded below subset of $\mathbb{R}_\mu$ has a $\mu-$ infimum.

Theorem 2. *Suppose the **Statement I** holds and A is a non $-$ empty subset of $\mathbb{R}_\mu$, which is bounded below in $\mathbb{R}_\mu$. Then A has an $\mu-$ infimum .*

Theorem 3. *Let A be a non $-$ empty subset of $\mathbb{R}_\mu$ and $\mu-$ bounded above in $\mathbb{R}_\mu$. An $\mu-$ upper bounded M of A is $\mu-$ supremum of A if and only if for each $\varepsilon > 0$ there exists an element $x_1 \varepsilon A$ such that $M - \varepsilon < x_1 \mu(x_1) \leq M$.*

§ 2. Sequence in $\mathbb{R}_\mu$:

Definition 5. *Let $\{x_n\}_n$ be a sequence in $\mathbb{R}_\mu$. Then $\{x_n\}_n$ is said to converge to x_0 in $\mathbb{R}_\mu$ if for every $\varepsilon > 0$, there exists a natural number k such that $|x_n - x_0|_\mu < \varepsilon$ for all $n \geq k$. If $\{x_n\}_n$ converges to x_0 in $\mathbb{R}_\mu$ then we express it by $x_n \underset{\mu}{\rightarrow} x_0$ as $n \longrightarrow \infty$, x_0 is called $\mu-$ limit of the sequence $\{x_n\}_n$ in $\mathbb{R}_\mu$ and $\{x_n\}_n$ is called $\mu-$ convergent in $\mathbb{R}_\mu$.*

Theorem 4. *If $\{x_n\}_n$ converges to l in $\mathbb{R}$ then $\{x_n\}_n$ converges to l in $\mathbb{R}_\mu$.*

Proof. Obvious □

Remark 1. *The converse of the above theorem is not necessarily true. In fact, $\mu-$ limit of $\{x_n\}_n$ is not necessarily unique. For example , let $x_n = logn + 1$ and $y_n = logn + \sqrt{2}$ for all $n \geq 1$, $\mu(x_n) = \mu(y_n) = \frac{n}{(n+1)^3}$, $\mu(x_n + x_m) = \mu(x_n - x_m) = \mu(x_n\, x_m) = \mu(x_n\, x_m^{-1}) = 1$, $\mu(y_n + y_m) = \mu(y_n - y_m) = \mu(y_n\, y_m) = \mu(y_n\, y_m^{-1}) = 1$, $\mu(x_n + y_m) = \mu(x_n - y_m) = \mu(x_n y_m^{-1}) = \mu(x_n^{-1} y_m^{-1}) = 1$, $\mu(x) = 0$ for all other $x \varepsilon \mathbb{R}$. Here we see that $x_n \underset{\mu}{\rightarrow} 0$ as $n \longrightarrow \infty$ but $\{x_n\}_n$ is not convergent in $\mathbb{R}$. We also see that $x_n \underset{\mu}{\rightarrow} 1 - \sqrt{2}$ as $n \longrightarrow \infty$.*

Remark 2. *If $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$ then $\{x_n\}_n$ converges to l in $\mathbb{R}$ if and only if $\{x_n\}_n$ converges to l in $\mathbb{R}_\mu$.*

Remark 3. *If $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$ and $x_n \underset{\mu}{\rightarrow} l$ as $n \longrightarrow \infty$ then l is unique .*

Theorem 5. *If* $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$ *and* $\{x_n\}_n$ *is* $\mu-$ *convergent in* $\mathbb{R}_\mu$ *then* $\{x_n\}_n$ *is* $\mu-$ *bounded in* $\mathbb{R}_\mu$.

Proof. Let $x_n \underset{\mu}{\rightarrow} l$ as $n \longrightarrow \infty$. $\Longrightarrow$ For $\varepsilon = 1$, there exists $k \varepsilon \mathbb{N}$ such that $|x_n - l|_\mu < 1$ for all $n \geq k \Longrightarrow l\,\mu(x_n - l) - 1 < x_n\,\mu(x_n - l) < l\,\mu(x_n - l) + 1$ for all $n \geq k \Longrightarrow a(la - 1) < x_n\,\mu(x_n) < \frac{1}{a}(l+1)$ for all $n \geq k \Longrightarrow \{x_n\,\mu(x_n) : n \varepsilon \mathbb{N}\}$ is bounded in $\mathbb{R} \Longrightarrow \{x_n\}_n$ is $\mu-$ bounded in $\mathbb{R}_\mu$. □

Remark 4. *If we drop the condition* $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$ *from the above theorem , the theorem may fails to hold. For example , consider* $x_n = e^n + 2$ *and define* $\mu(x_n) = 1$, $\mu(x_n + 1) = \frac{1}{(e^n+1)^2}$ *for all* $n \geq 1$, $\mu(x_n + x_m) = \mu(x_n - x_m) = \mu(x_n x_m) = \mu(x_n x_m^{-1}) = 1$, $\mu(x) = 0$ *for all other* $x \varepsilon \mathbb{R}$. *Here ,* $x_n \underset{\mu}{\rightarrow} 1$ *as* $n \longrightarrow \infty$ *but* $\{x_n\}_n$ *is not* $\mu-$ *bounded in* $\mathbb{R}_\mu$.

Theorem 6. *Let* $x_n \longrightarrow l$ *and* $y_n \longrightarrow m$ *as* $n \longrightarrow \infty$. *Then* $x_n + y_n \underset{\mu}{\rightarrow} l + m$ *as* $n \longrightarrow \infty$

Proof. Obvious □

Note 1. $x_n \underset{\mu}{\rightarrow} l$, $y_n \underset{\mu}{\rightarrow} m$ *as* $n \longrightarrow \infty$ *do not always imply* $x_n + y_n \underset{\mu}{\rightarrow} l + m$ *as* $n \longrightarrow \infty$. *For example, let* $x_n = y_n = \frac{(n+1)^2}{n^2}$, $\mu(x_n - 1) = \frac{n^2}{(2n+1)^3}$, $\mu(2x_n) = \frac{n^2}{2(n+1)^3}$, $\mu(t_n + t_m) = \mu(t_n - t_m) = \mu(t_n t_m) = \mu(t_n t_m^{-1}) = 1$, *where* $t_n = x_n - 1$, $m \neq n$, $\mu(x) = 0$ *for all other* $x \varepsilon \mathbb{R}$. *Here we see that* $x_n \underset{\mu}{\rightarrow} 1$, $y_n \underset{\mu}{\rightarrow} 1$ *as* $n \longrightarrow \infty$. *But* $x_n + y_n \underset{\mu}{\rightarrow} 0$ *as* $n \longrightarrow \infty$.

Theorem 7. *If* $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$, $x_n \underset{\mu}{\rightarrow} l$, $y_n \underset{\mu}{\rightarrow} m$ *as* $n \longrightarrow \infty$ *then* $x_n + y_n \underset{\mu}{\rightarrow} l + m$ *as* $n \longrightarrow \infty$.

Theorem 8. *Let* $x_n \longrightarrow l$ *and* $y_n \longrightarrow m$ *as* $n \longrightarrow \infty$. *Then* $x_n y_n \underset{\mu}{\rightarrow} lm$ *as* $n \longrightarrow \infty$

Note 2. $x_n \underset{\mu}{\rightarrow} l$, $y_n \underset{\mu}{\rightarrow} m$ *as* $n \longrightarrow \infty$ *do not always imply* $x_n y_n \underset{\mu}{\rightarrow} lm$ *as* $n \longrightarrow \infty$. *For example, let* $x_n = (1 + \frac{1}{n})^2$ *and* $\mu(x_n - 1) = \frac{n^2}{(2n+1)^3}$; $y_n = \frac{n+1}{3n+1}$ *and* $\mu(y_n - \frac{1}{3}) = \frac{3(3n+1)}{2n^2}$; $\mu(x_n y_n) = \frac{1}{n}$ *and define* μ *suitably at all other points of* $\mathbb{R}$ *such that all the conditions of the definition*(1) *are being satisfied. Here we see that* $x_n \underset{\mu}{\rightarrow} 1$, $y_n \underset{\mu}{\rightarrow} \frac{1}{3}$ *as* $n \longrightarrow \infty$. *But* $x_n y_n \underset{\mu}{\rightarrow} 0$ *as* $n \longrightarrow \infty$.

Theorem 9. *If* $inf\{\mu(x) : x \varepsilon \mathbb{R}\} = a > 0$, $x_n \underset{\mu}{\rightarrow} l$, $y_n \underset{\mu}{\rightarrow} m$ *as* $n \longrightarrow \infty$ *then* $x_n y_n \underset{\mu}{\rightarrow} l m$ *as* $n \longrightarrow \infty$.

Definition 6. *A sequence* $\{x_n\}_n$ *in* $\mathbb{R}_\mu$ *is said to be* $\mu-$ *increasing if* $x_1 \leq_\mu x_2 \leq_\mu \cdots \leq_\mu x_n \leq_\mu x_{n+1} \leq_\mu \cdots$

Theorem 10. *If* $\{x_n\}_n$ *is* $\mu-$ *increasing and* $\mu-$ *bounded above then* $\{x_n \mu(x_n)\}_n$ *converges to its supremum .*

§ 3 $\mu-$ **Fuzzy Complex Numbers :**

Definition 7. *Let* $z \varepsilon \mathbb{C}_\mu$. $\mu-$ *conjugate of* z *is denoted by* $\overline{z}_\mu$ *and defined by* $\overline{z}_\mu = \overline{z}\mu(z)$, *where* $\overline{z}$ *is the usual complex conjugate of* z .

Property 4. (i) $\overline{(\overline{z}_\mu)}_\mu = z\,\mu(z)\,\mu(\overline{z}_\mu)$

(ii) $\frac{\overline{(z_1+z_2)}_\mu}{\mu(z_1+z_2)} = \frac{\overline{(z_1)}_\mu}{\mu(z_1)} + \frac{\overline{(z_2)}_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 + z_2) \neq 0$

(iii) $\frac{\overline{(z_1-z_2)}_\mu}{\mu(z_1-z_2)} = \frac{\overline{(z_1)}_\mu}{\mu(z_1)} - \frac{\overline{(z_2)}_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 - z_2) \neq 0$

(iv) $\frac{\overline{(z_1 z_2)}_\mu}{\mu(z_1 z_2)} = \frac{\overline{(z_1)}_\mu}{\mu(z_1)} \frac{\overline{(z_2)}_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 z_2) \neq 0$

(v) $\frac{\overline{(z_1/z_2)}_\mu}{\mu(z_1/z_2)} = \frac{\overline{(z_1)}_\mu}{\overline{(z_2)}_\mu} \frac{\mu(z_2)}{\mu(z_1)}$, $\mu(z_1), \mu(z_2), \mu(z_1/z_2) \neq 0$

(vi) $z\,\mu(z) + \overline{z}_\mu = 2\,Re(z)\,\mu(z)$

(vii) $z\,\mu(z) - \overline{z}_\mu = 2\,Im(z)\,\mu(z)$

Definition 8. *Let* $z \varepsilon \mathbb{C}_\mu$. $\mu-$ *modulus of* z *is denoted by* $|z|_\mu$ *and defined by* $|z|_\mu = |z|\mu(z)$, *where* $|z|$ *is the usual modulus of* z.

Property 5. (i) $\frac{|z_1 z_2|_\mu}{\mu(z_1 z_2)} = \frac{|z_1|_\mu}{\mu(z_1)} \frac{|z_2|_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 z_2) \neq 0$

(ii) $\frac{|z_1+z_2|_\mu}{\mu(z_1+z_2)} \leq \frac{|z_1|_\mu}{\mu(z_1)} + \frac{|z_2|_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 + z_2) \neq 0$

(iii) $\frac{|z_1/z_2|_\mu}{\mu(z_1/z_2)} = \frac{|z_1|_\mu}{|z_2|_\mu} \frac{\mu(z_2)}{\mu(z_1)}$, $\mu(z_1), \mu(z_2), \mu(z_1/z_2) \neq 0$

(iv) $|\overline{z}_\mu| = |z|\mu(z)$

(v) $\frac{|z_1-z_2|_\mu}{\mu(z_1-z_2)} \geq \frac{|z_1|_\mu}{\mu(z_1)} - \frac{|z_2|_\mu}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 - z_2) \neq 0$

(vi) $|z|_\mu \geq Re(z)\,\mu(z)$ *and* $|z|_\mu \geq Im(z)\,\mu(z)$

(vii) $z\,\overline{z}_\mu = z\,\overline{z}\,\mu(z) = |z|^2\,\mu(z)$

Definition 9. *Let* $z \varepsilon \mathbb{C}_\mu$. $\mu-$ *argument of* z *is denoted by* $\arg_\mu(z)$ *and defined by* $\arg_\mu(z) = \arg(z)\,\mu(z)$, $-\pi < \arg(z) \leq \pi$.

Property 6. $\frac{\arg_\mu(z_1 z_2)}{\mu(z_1 z_2)} = \frac{\arg_\mu(z_1)}{\mu(z_1)} + \frac{\arg_\mu(z_2)}{\mu(z_2)} + 2k\pi$, *where* $k = 0$ *if* $-\pi < \arg(z_1) + \arg(z_2) \leq \pi$; $k = 1$ *if* $\arg(z_1) + \arg(z_2) \leq -\pi$; $k = -1$ *if* $\arg(z_1) + \arg(z_2) > \pi$, $\mu(z_1), \mu(z_2), \mu(z_1 z_2) \neq 0$.

Definition 10. *Let* $z \varepsilon \mathbb{C}_\mu$. *Then* $\exp_\mu(z) = \exp(z)\,\mu(z)$, *where* $\exp(z)$ *denotes the usual exponential function in* $\mathbb{C}$.

Property 7. (i) $\frac{\exp_\mu(z_1 + z_2)}{\mu(z_1 + z_2)} = \frac{\exp_\mu(z_1)}{\mu(z_1)} \frac{\exp_\mu(z_2)}{\mu(z_2)}$ *provided* $\mu(z_1), \mu(z_2)$, $\mu(z_1 + z_2) \neq 0$

(ii) $\frac{\exp_\mu(z_1 - z_2)}{\mu(z_1 - z_2)} = \frac{\exp_\mu(z_1)}{\exp_\mu(z_2)} \frac{\mu(z_2)}{\mu(z_1)}$, $\mu(z_1), \mu(z_2), \mu(z_1 - z_2) \neq 0$

Note 3. $\exp_\mu(0) = \exp(0)\,\mu(0) = 1$

Note 4. $\left(\frac{exp_\mu(z)}{\mu(z)} \right)^n = \frac{exp_\mu(nz)}{\mu(nz)}$, $\mu(z), \mu(nz) \neq 0$

Definition 11. *Let* $z \varepsilon \mathbb{C}_\mu$ *and* $z \neq 0$. $\mu-$ *logarithm of* z *is denoted by* $Log_\mu(z)$ *and defined by* $Log_\mu(z) = Log(z)\,\mu(z)$.

Property 8. (i) $\frac{Log_\mu(z_1 z_2)}{\mu(z_1 z_2)} = \frac{Log_\mu(z_1)}{\mu(z_1)} + \frac{Log_\mu(z_2)}{\mu(z_2)}$, $\mu(z_1), \mu(z_2)$, $\mu(z_1 z_2) \neq 0$

(ii) $\frac{Log_\mu(z_1 / z_2)}{\mu(z_1 / z_2)} = \frac{Log_\mu(z_1)}{\mu(z_1)} - \frac{Log_\mu(z_2)}{\mu(z_2)}$, $\mu(z_1), \mu(z_2), \mu(z_1 / z_2) \neq 0$

Definition 12. *Let* $a \varepsilon \mathbb{C}_\mu$, $a \neq 0$ *and* $z \varepsilon \mathbb{C}_\mu$. *We now define* a_μ^z *by* $a_\mu^z = a^z \mu(z) = \exp_\mu(z\, Log\, a)$ *and* $p.v.(a_\mu^z) = p.v.(a^z)\,\mu(z)$

Property 9. (i) $\frac{p.v.(a_\mu^{z_1 + z_2})}{\mu(z_1 + z_2)} = \frac{p.v.(a_\mu^{z_1})}{\mu(z_1)} + \frac{p.v.(a_\mu^{z_2})}{\mu(z_2)}$, $\mu(z_1), \mu(z_2)$, $\mu(z_1 + z_2) \neq 0$

(ii) $(ab)_\mu^z\, \mu(z) = a_\mu^z b_\mu^z$

Conclusion : In next research, we are trying to established the concepts of a function in $\mathbb{R}_\mu$ or $\mathbb{C}_\mu$, limit , continuity , differentiability etc. of such function .

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